

Exercise sheet 4

Exercise 5:

Find two functions f and g (both of type $\mathbb{N} \rightarrow \mathbb{N}$) such that neither $f(n) = O(g(n))$ nor $f(n) = \Omega(g(n))$. Show that your claim is correct!

Exercise 5a:

Given be the function $f(n) = n^3 - 3n + 10$

- Define a non-tight asymptotic upper bound $o(g(n))$ for $f(n)$!
- Give a formal justification using the definition of non-tight asymptotic upper bound!
- Define an asymptotic upper bound and an asymptotic lower bound for $f(n)$ that is also a tight bound !
- Give a formal justification using the definition of asymptotic tight bound!

Exercise 5b:

Prove by using the rules for Landau notation that the following equation holds: $4n^3 - 100n + 1500 = \Theta(n^3 + 2n^2 + 3n + 4)$

Hint: Do not use the definition of Θ , but use the fact that polynomials are bounded asymptotically tight by n to the highest power of the polynomial.

Exercise 6:

Illustrate how the algorithm Insertion_sort works on the input sequence $\langle 77, 16, 35, 37, 100, 20, 59 \rangle$!

Exercise 7:

Let $f(n) = \log(n!)$. Show that $f(n) = O(n \log n)$ and $f(n) = \Omega(n)$.

In the exercise class an improved asymptotic lower bound for $f(n)$ will be shown ($f(n) = \Omega(n \log n)$). Assuming this result had already been proved: What is the asymptotic growth of $f(n)$?