

Exercise sheet 3

Exercise 3:

Let f_i be the i -th Fibonacci number ($i = 0, 1, 2, \dots$).

Let $\phi = \frac{1+\sqrt{5}}{2}$ and $\hat{\phi} = \frac{1-\sqrt{5}}{2}$

(ϕ is known as the golden ratio and $\hat{\phi}$ as its conjugate). Prove by induction that

$$f_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} !$$

Exercise 4:

a) Show that $\frac{1}{3}n^3 - 2n^2 + 5n - 2 = \Theta(n^3)$.

b) Show that for any constants $a, b \in \mathbb{R}$, where $b > 0$, we have $(n+a)^b = \Theta(n^b)$.