Using Optimization in Smart Grid Applications

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Why Optimization

- Smart Grid allows better coordination between the different entities
- Optimization can be used to find the best strategy, size of different components,…
- We are not going to focus on the optimization algorithms rather than on how to use them
- There are a lot of tools that can be used
- We are going to use some tools to solve the problems.

Several tools

- ADMB
- CONDOR
- Joptimizer
- NLOPT

Nlopt

+

What is Optimization?

- **Optimization** is the mathematical discipline which is concerned with finding the maxima and minima of functions, possibly subject to constraints.

What do we optimize?

- A real function of n variables

$$f(x_1, x_2, \ldots, x_n)$$

- with or without constraints

Unconstrained optimization

$$\min \; f(x,y) = x^2 + 2y^2$$

Optimization with constraints

$$\min \ f(x,y) = x^2 + 2y^2$$

$$x > 0$$

or

$$\min \ f(x,y) = x^2 + 2y^2$$

$$-2 < x < 5, y \geq 1$$

or

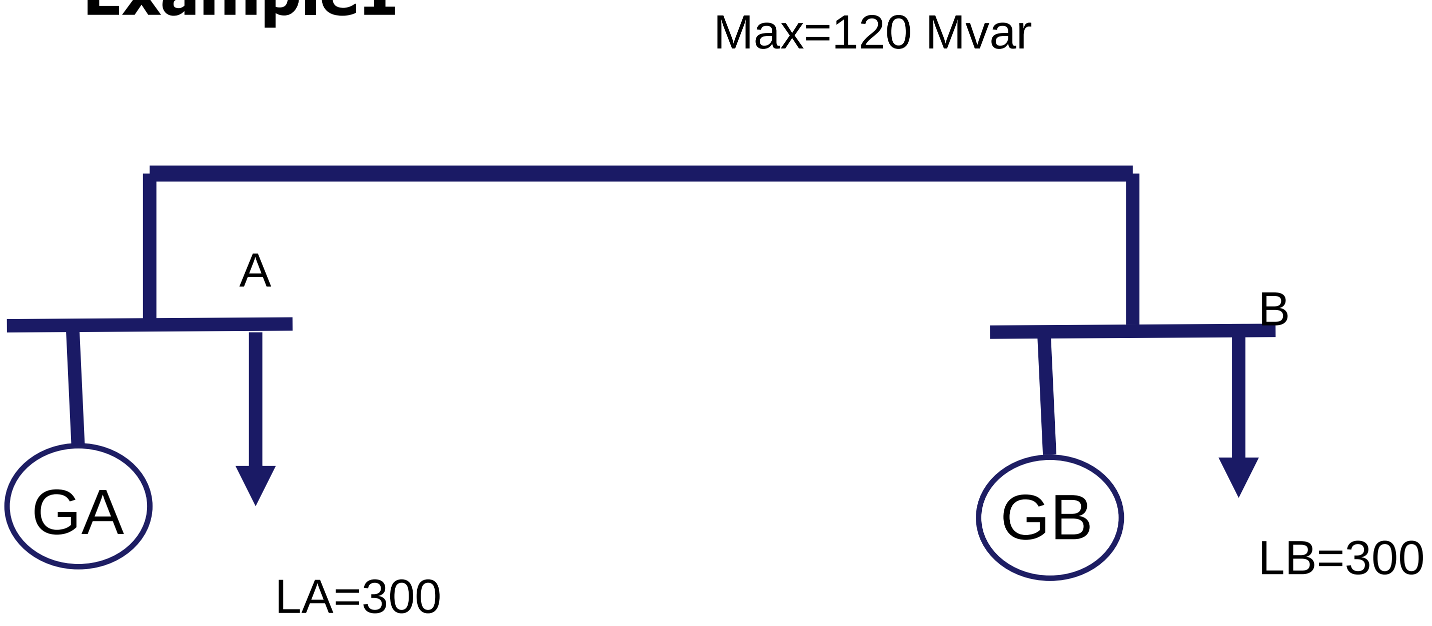
$$\min \ f(x,y) = x^2 + 2y^2$$

$$x + y = 2$$

Nloptr Package

- We are going to use a package called nloptr to solve non-linear optimization problems.

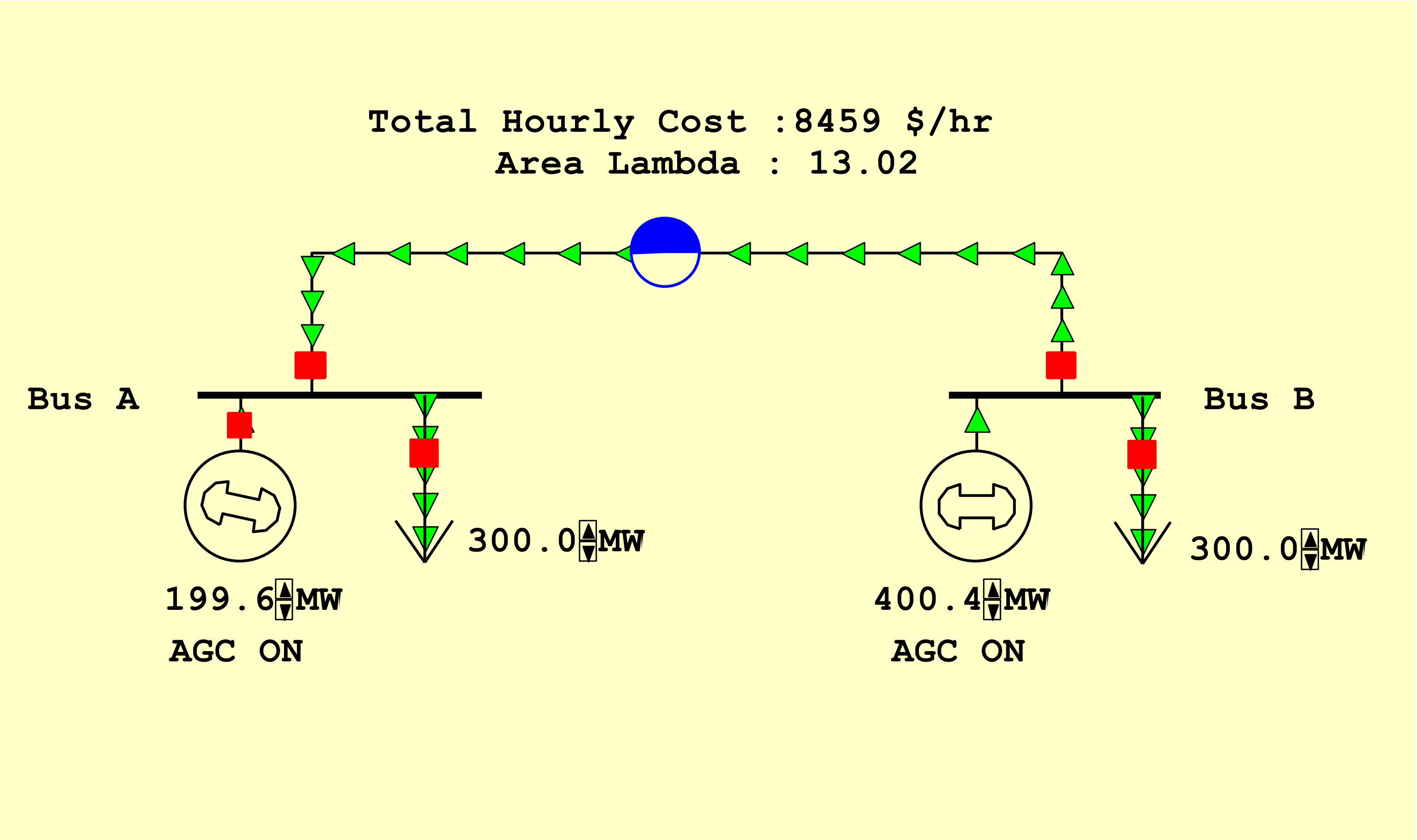
Example1



$$C_A(PG_A) = 399.8 + 11.69PG_A + 0.00334PG_A^2 \text{ Euro/hr}$$

$$C_B(PG_B) = 616.9 + 11.83PG_B + 0.00149PG_B^2 \text{ Euro/hr}$$

Example1



Total Hourly Cost :8459 $/hr
Area Lambda : 13.02
Bus A
Bus B
300.0 MW
300.0 MW
199.6 MW
AGC ON
400.4 MW
AGC ON

Generator cost curves

$$C_A(PG_A) = 399.8 + 11.69PG_A + 0.00334PG_A^{\;2} \text{ Euro/hr}$$

$$C_B(PG_B) = 616.9 + 11.83PG_B + 0.00149PG_B^{\;2} \text{ Euro/hr}$$

- PGA+PGB=600

Generator cost curves

- How much each generator should produce to cover the 600MW demand
- The Problem can be written as:
- **MINIMIZE** (C_A(PGA)+C_B(PGB))
- Subject to
- $PG_A+PG_B=600$
- $PG_A>=0$
- $PG_B>=0$

Code

```
#Copyright (C) 2013-2015 Abdalkarim Awad
#if the package is not yet installed, uncomment the following
#line to install the nonlinear optimization package
#install.packages("nloptr")
library('nloptr') #use the library

#objective function
#It is the sum of all costs of all generators
#let PA=x1, PB=x2
#f(x)=399.8+11.69*x[1]+0.00334*x[1]^2+616.9+11.83*x[2]+0.00149*x[2]^2
function_f <- function(x){
return(399.8+11.69*x[1]+0.00334*x[1]^2+616.9+11.83*x[2]+0.00149*x[2]^2 )
}
# constraint function g(x),
#x1+x2=600 ==> x1+x2-600=0
#the optimization method COBYLA supports equality constraints
#by transforming them into two inequality constraints.
#x1+x2-600=0 ==> x1+x2-600<=0 and x1+x2-600>=0
#==> x1+x2-600<=0  and -x1-x2+600<=0
const_g<- function( x) {
    return( rbind(c(x[1]+x[2]-600), -c(+x[1]+x[2]-600)) )
}
```

Code

```
# Solve using NLOPT_LN_COBYLA
results <- nloptr( x0=c(100,100), #initial values
    eval_f=function_f,  #min (f(x))
    lb = c(0,0), #lower bound for x1 and x2
    ub = c(Inf,Inf), #upper bound for x1 and x2
    eval_g_ineq= const_g, #equality and inequality constraints
    opts = list("algorithm"="NLOPT_LN_COBYLA",xtol_rel = 1e-4)) #options
print( results )
min_value=results$objective
x=results$solution
PA=x[1]
PB=x[2]
min_value
PA
PB
```

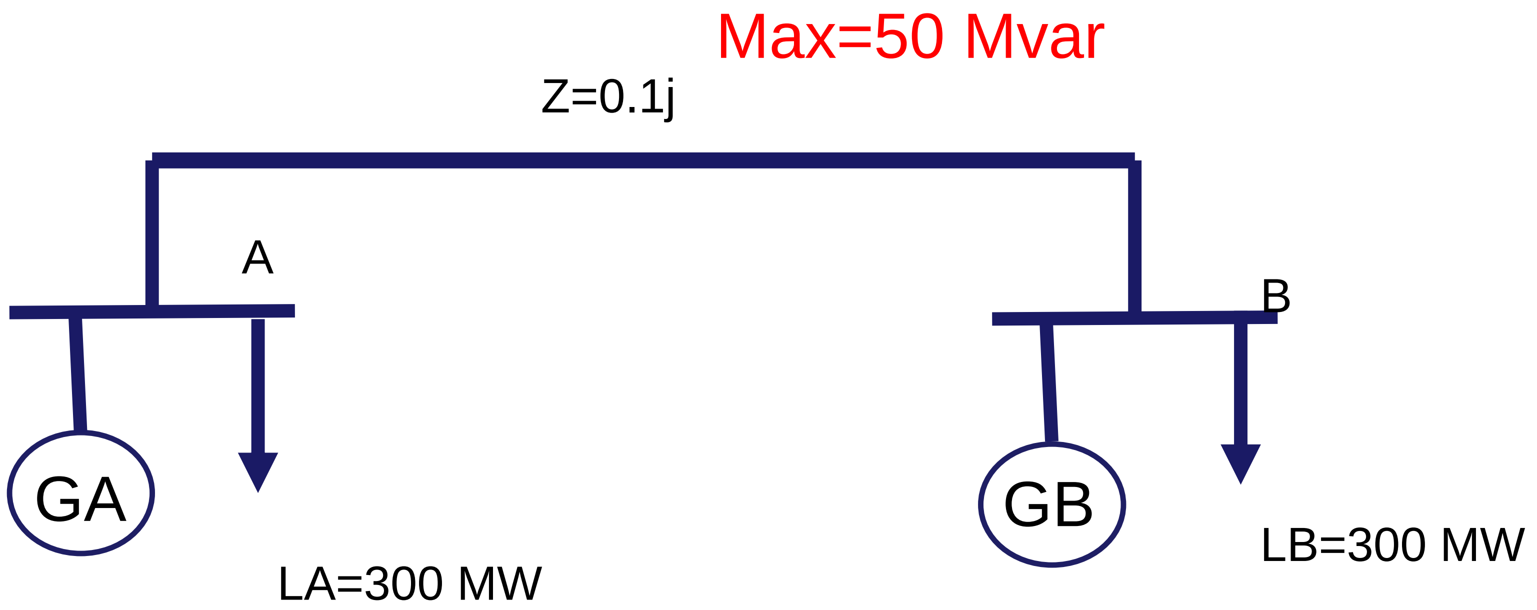
Run

```
Minimization using NLopt version 2.4.0

NLopt solver status: 4 ( NLOPT_XTOL_REACHED: Optimization stopped because
xtol_rel or xtol_abs (above) was reached. )

Number of Iterations....: 36
Termination conditions:  xtol_rel: 1e-04
Number of inequality constraints:  2
Number of equality constraints:    0
Optimal value of objective function:  8458.69917197555
Optimal value of controls: 199.5807 400.4193


> min_value=results$objective
> x=results$solution
> PA=x[1]
> PB=x[2]
> min_value
[1] 8458.699
> PA
[1] 199.5807
> PB
[1] 400.4193
>
>
```

Example2

- Add constraints to generator 2
- $0<PGB<250$
- What is the total costs now?

Example3



Write R script to find P1 and P2 that minimizes the operation costs and takes into considerations line limitations

Including power Flow

$$P_i = \sum_{k=1}^{N} (B_{ik}(\theta_i - \theta_k))$$

P1=(B12)(θ1-θ2)

Z=0.1j ➔ y=-10j➔B=-10

P1=(B12)θ1-B12(θ2)

P2=B21(θ2- θ1)

P2=-B21(θ1)+(B21)(θ2)

$$\begin{bmatrix} PA-3 \\ PB-3 \end{bmatrix} = \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix} \begin{bmatrix} \theta 1 \\ \theta 2 \end{bmatrix}$$

$$\begin{bmatrix} PA-3 \\ PB-3 \end{bmatrix} = \begin{bmatrix} -10 & 10 \\ 10 & -10 \end{bmatrix} \begin{bmatrix} \theta 1 \\ \theta 2 \end{bmatrix}$$

Solve with respect to bus #1. i.e $\theta 1=0$ (Slack bus)

PB-3=-(10)θ2➔

PB-3+(10)(θ2)=0

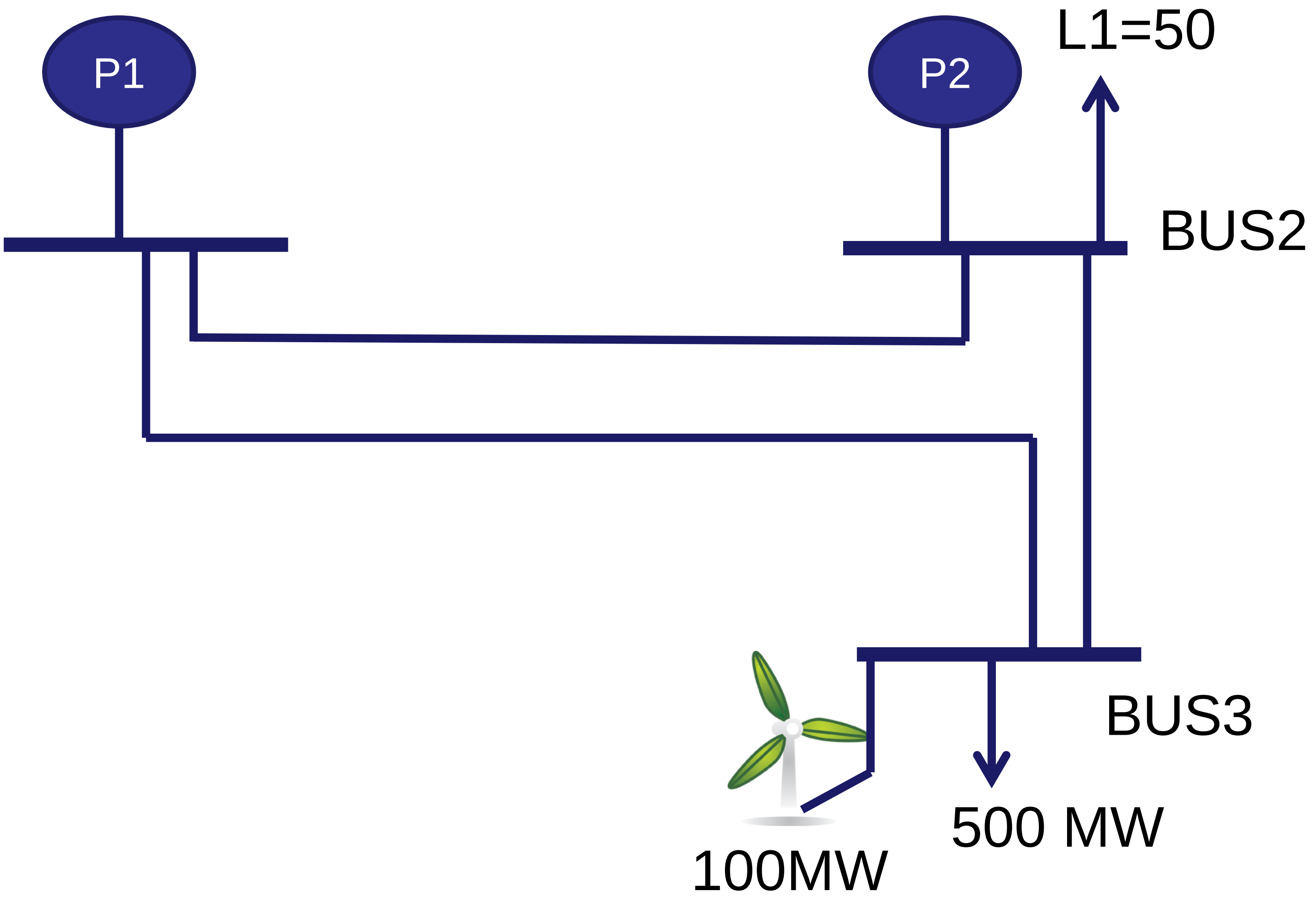
P12=B12*(θ1-θ2)

|P12|=|10(θ1-θ2)|<0.5

-0.5<10*θ2<0.5

-0.05<θ2<0.05

Example 4



Generator cost curves

- $C1(P1) = 400 + 9 \times P1 + 0.0015 \times (P1)^2$

$100 \text{ MW} \leq P1 \leq 600 \text{ MW}$

- $C3(P2) = 100 + 8 \times P2 + 0.0048 \times (P2)2$

$50 \text{ MW} \leq P2 \leq 400 \text{ MW}$

Minimize $(C1(P1)+C3(P2))$

Optimization Problem

- Minimize ($C1(P1)+C2(P2)$)
- Subject to:
- $100\ MW \le P1 \le 600\ MW$
- $50\ MW \le P2 \le 400\ MW$

Solution

- $9 + 2*0.0015 \times (P1)=\lambda$
- $8+ 2*0.0048 \times (P2) =\lambda$

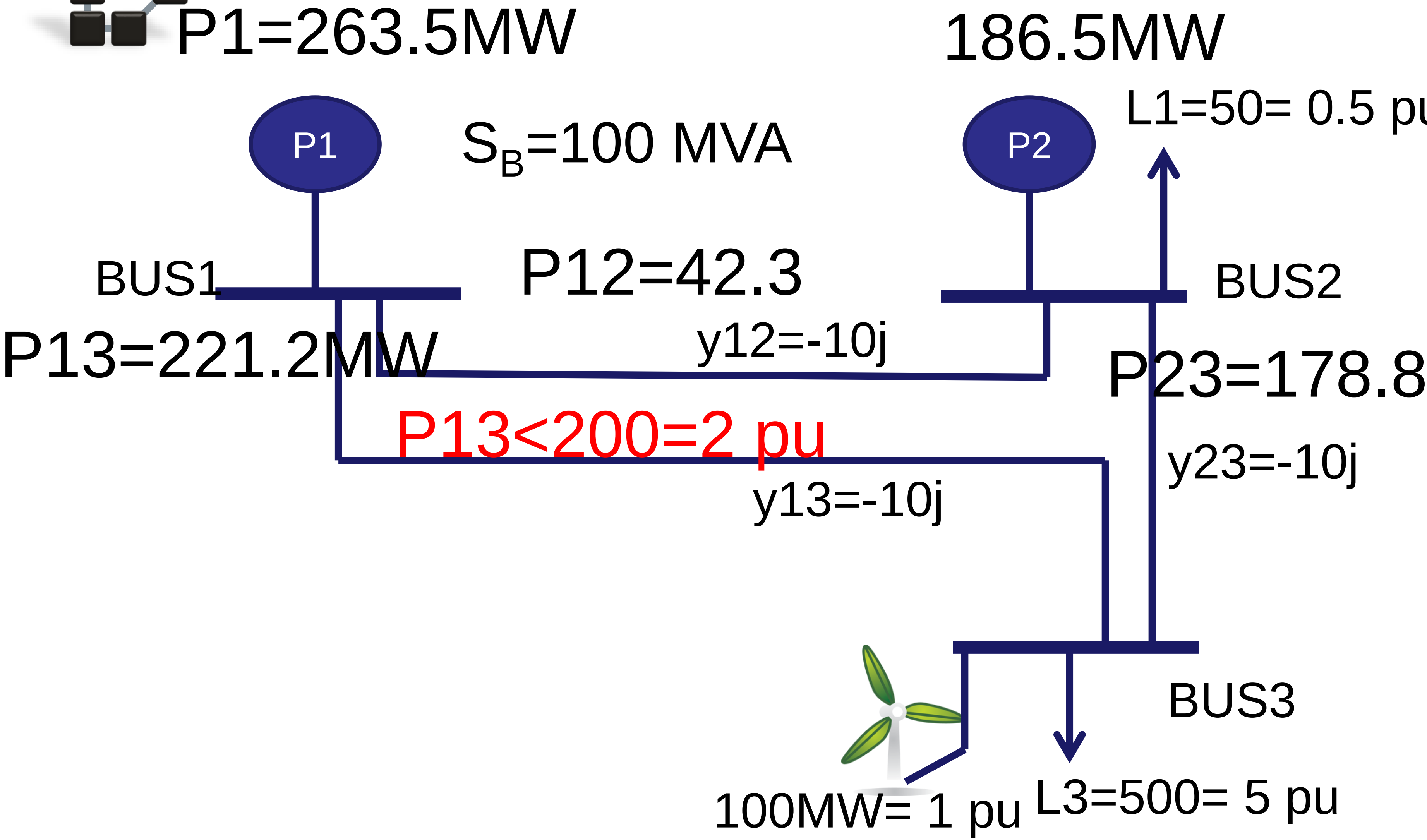
P1+P2=500+50-100=450

➔ P1=263.5

➔ P2=186.5

Minimum Cost = **4634.60**

Do they satisfy all constraints?


P1=263.5MW
186.5MW
L1=50= 0.5 pu
P1
P2
S_B=100 MVA
BUS1
P12=42.3
BUS2
P13=221.2MW
y12=-10j
P23=178.8
P13<200=2 pu
y23=-10j
y13=-10j
BUS3
100MW= 1 pu
L3=500= 5 pu

Including power Flow

$$P_i = \sum_{k=1}^{N} (B_{ik}(\theta_i - \theta_k))$$

P1=(B12)(θ1-θ2)+B13(θ1-θ3

➔

P1=(B12+B13)θ1-B12(θ2)-B13(θ3)

P2=-B21(θ1)+(B21+B23)(θ2)-B23(θ3)

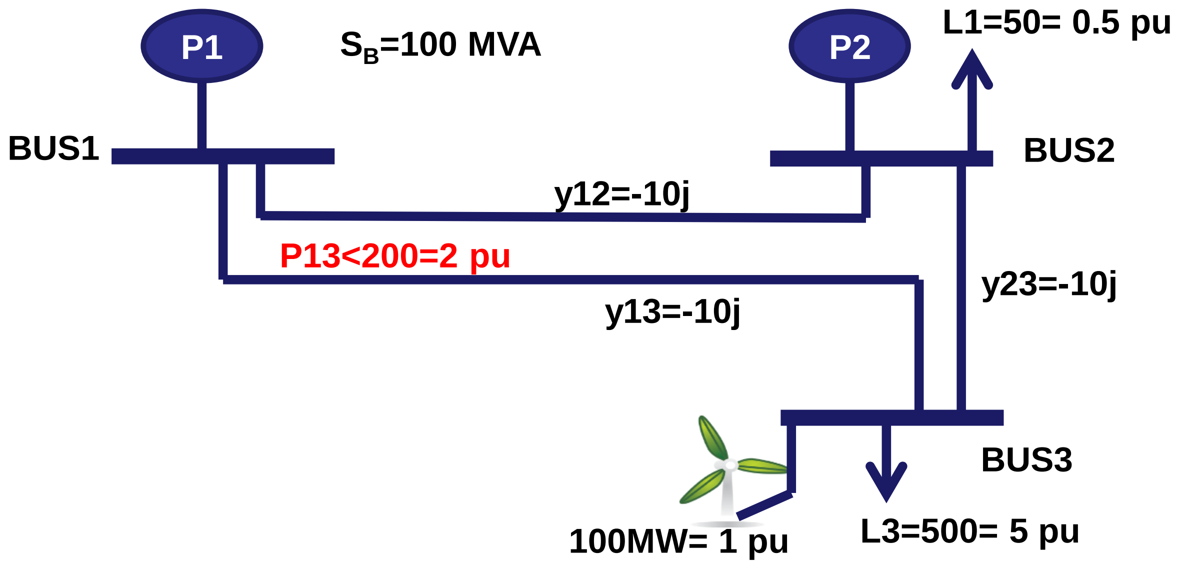
P3=-B31(θ1)-B32(θ2)+(B31+B32)(θ3)

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} \begin{bmatrix} B12+B13 & -B12 & -B13 \\ -B21 & B21+B23 & -B23 \\ -B31 & -B32 & B21+B32 \end{bmatrix} \begin{bmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{bmatrix}$$

Compare to Y matrix!

$$\begin{bmatrix} P1 \\ P2-0.5 \\ 1-5 \end{bmatrix} = \begin{bmatrix} -20 & 10 & 10 \\ 10 & -20 & 10 \\ 10 & 10 & -20 \end{bmatrix} \begin{bmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{bmatrix}$$

singular



$$\begin{bmatrix} P1 \\ P2-0.5 \\ -4 \end{bmatrix} = \begin{bmatrix} -20 & 10 & 0 \\ 10 & -20 & 10 \\ 0 & 10 & -20 \end{bmatrix} \begin{bmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{bmatrix}$$

Solve with respect to bus #1. i.e $\theta 1=0$ (Slack bus)

$$\begin{bmatrix} P2-0.5 \\ -4 \end{bmatrix} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix} \begin{bmatrix} \theta 2 \\ \theta 3 \end{bmatrix}$$

P13<200 ➔ B13($\theta 1-\theta 3$)<2 (pu)➔ $\theta 3<0.20$ (B13=-10)

Optimization Problem

- Minimize (C1(P1)+C2(P2)) (in pu)
- Subject to:
- 1 pu ≤ P1 ≤ 6 pu
- 0.5 pu ≤ P2 ≤ 4 pu
- P1+P2=4

$$\begin{bmatrix} P2-0.5 \\ -4 \end{bmatrix} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix} \begin{bmatrix} \theta 2 \\ \theta 3 \end{bmatrix}$$

$\Theta 3 < 0.20$

Solution (using R)

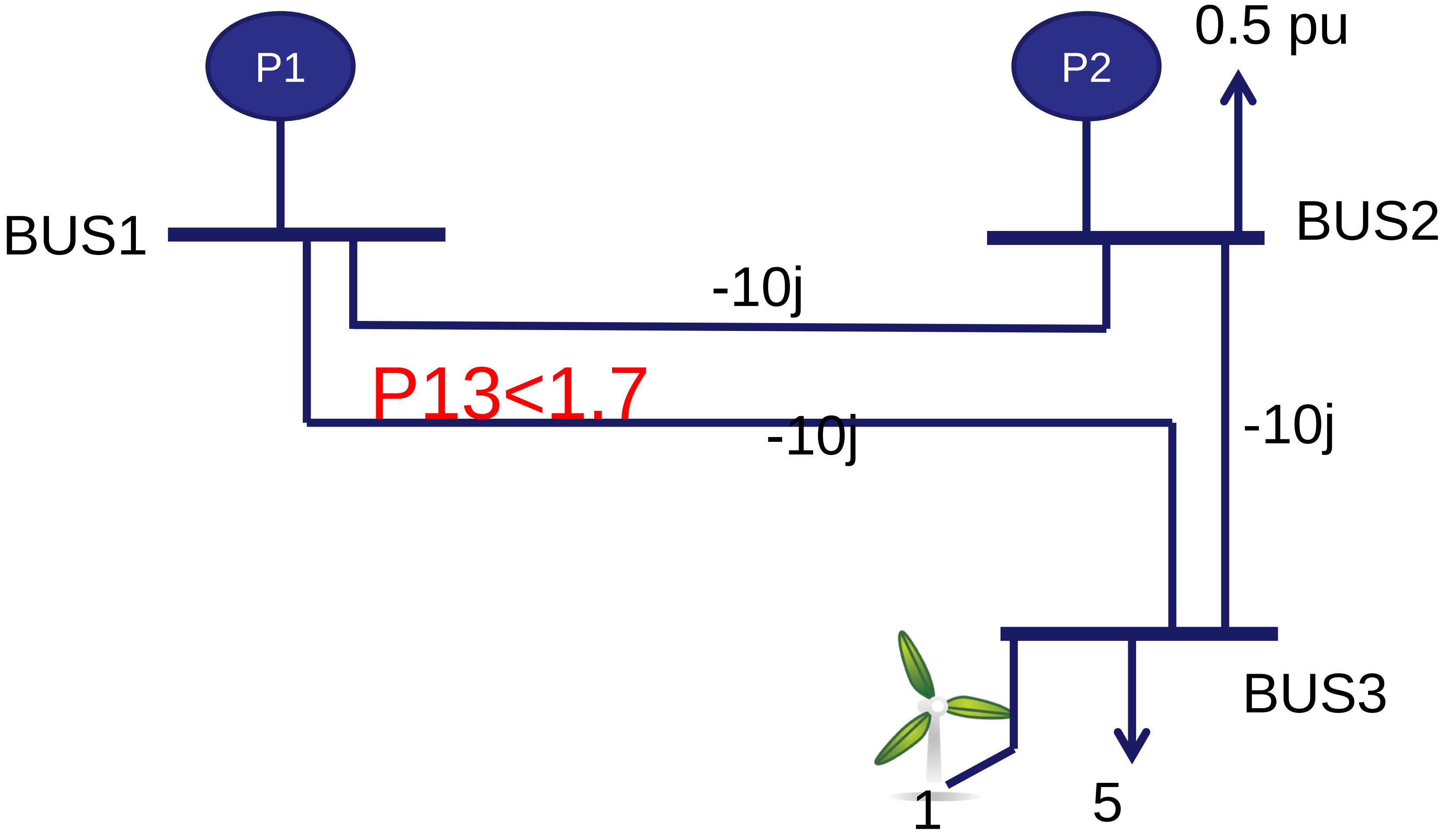
$$\begin{bmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}$$

P1=200 MW

P2=250 MW

P13=-B13(θ1-θ3)=-10*(0-0.2)=2 pu➔=200 MW

Costs=**4659.95>4634.60** Euro


P1
P2
0.5 pu
BUS1
BUS2
-10j
P13<1.7
-10j
-10j
BUS3
1
5

Optimization Problem

- Minimize (C1(P1)+C2(P2)) (in pu)
- Subject to:
- 1 pu ≤ P1 ≤ 6 pu
- 0.5 pu ≤ P2 ≤ 4 pu

$$\begin{bmatrix} P2 \\ -4 \end{bmatrix} = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix} \begin{bmatrix} \theta 2 \\ \theta 3 \end{bmatrix}$$

P13<170 ➔ B13(θ1-θ3)<1.70➔ θ3<0.17

Solution (using R)

P1=110

P2=340

θ2=-0.06

θ3=0.17

- The total generation cost becomes: **4783.03> 4634.60**
- Here, we had to sacrifice “cost” for “implementation”.