Exercise1

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Informatik 7
Rechnernetze und
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Review of Phasors

Goal of phasor analysis is to simplify the analysis of constant frequency ac systems

$$v(t) = V_{max} \cos(\omega t + \theta_V)$$
$$i(t) = I_{max} \cos(\omega t + \theta_I)$$

Root Mean Square (RMS) voltage of sinusoid

$$\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{V_{max}}{\sqrt{2}}$$

Phasor Representation

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

Phasor notation is developed by rewriting using Euler's identity

$$v(t) = \sqrt{2}\left|V\right|\cos(\omega t + \theta_V)$$

$$v(t) = \sqrt{2}\left|V\right|\mathrm{Re}\left[e^{j(\omega t + \theta_V)}\right]$$

(Note: $\left|V\right|$ is the RMS voltage)

Phasor Representation, cont'd

The RMS, cosine-referenced voltage phasor is:

$$V = |V| e^{j\theta_V} = |V| \angle \theta_V$$

$$v(t) = \operatorname{Re} \sqrt{2}\, V e^{j\omega t} e^{j\theta_V}$$

$$V = |V| \cos\theta_V + j|V| \sin\theta_V$$

$$I = |I| \cos\theta_I + j|I| \sin\theta_I$$

(Note: Some texts use "boldface" type for complex numbers, or "bars on the top")

Advantages of Phasor Analysis

Device	Time Analysis	Phasor
Resistor	$v(t) = Ri(t)$	$V = RI$
Inductor	$v(t) = L\dfrac{di(t)}{dt}$	$V = j\omega LI$
Capacitor	$\dfrac{1}{C}\int_0^t i(t)dt + v(0)$	$V = \dfrac{1}{j\omega C}I$

$$Z = \text{Impedance} = R + jX = \left|Z\right|\angle\phi$$

$$R = \text{Resistance}$$

$$X = \text{Reactance}$$

$$\left|Z\right| = \sqrt{R^2 + X^2} \qquad \phi = \arctan(\frac{X}{R})$$

(Note: Z is a complex number but not a phasor)

Example1

- A 50-Hz, single-phase source with $230\angle 30$ volts is applied to a circuit element.
- A)Determine the instantaneous source voltage. Also determine the phasor and instantaneous currents entering the positive terminal if the circuit element is
- B) a 20-ohm resistor,
- C) a 10-mH inductor,
- D) a capacitor with 25 ohm reactance.

Complex Power

Power

$$p(t) = v(t)\, i(t)$$

$$\text{v(t)} = V_{\max} \cos(\omega t + \theta_V)$$

$$i(\text{t}) = I_{\max} \cos(\omega t + \theta_I)$$

$$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$p(t) = \frac{1}{2} V_{\max} I_{\max} [\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

Complex Power, cont'd

Average Power

$$p(t) = \frac{1}{2}V_{\max}I_{\max}[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt$$

$$= \frac{1}{2}V_{\max}I_{\max}\cos(\theta_V - \theta_I)$$

$$= |V||I|\cos(\theta_V - \theta_I)$$

Power Factor Angle $= \phi = \theta_V - \theta_I$

Complex Power

$$
\begin{aligned}
S &= |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)] \\
&= P + jQ \\
&= V\, I^*
\end{aligned}
$$

(Note: S is a complex number but not a phasor)

P = Real Power (W, kW, MW)

Q = Reactive Power (var, kvar, Mvar)

S = Complex power (VA, kVA, MVA)

Power Factor (pf) = $\cos\phi$

If current leads voltage then pf is leading

If current lags voltage then pf is lagging

Complex Power, cont'd

Relationships between real, reactive and complex power

$$P = |S|\cos\phi$$

$$Q = |S|\sin\phi = \pm|S|\sqrt{1-pf^2}$$

Example: A load draws 100 kW with a leading pf of 0.85. What are ϕ (power factor angle), Q and $|S|$?

$$\phi = -\cos^{-1}0.85 = -31.8°$$

$$|S| = \frac{100kW}{0.85} = 117.6 \text{ kVA}$$

$$Q = 117.6\sin(-31.8°) = -62.0 \text{ kVar}$$

Power Consumption in Devices

Resistors only consume real power

$$P_{\text{Resistor}} = \left| I_{\text{Resistor}} \right|^2 R$$

Inductors only consume reactive power

$$Q_{\text{Inductor}} = \left| I_{\text{Inductor}} \right|^2 X_{\text{L}}$$

Capacitors only generate reactive power

$$Q_{\text{Capacitor}} = -\left| I_{\text{Capacitor}} \right|^2 X_{\text{C}} \qquad X_C = \frac{1}{\omega C}$$

$$Q_{\text{Capacitor}} = -\frac{\left| V_{\text{Capacitor}} \right|^2}{X_{\text{C}}}$$ (Note-some define X_C negative)

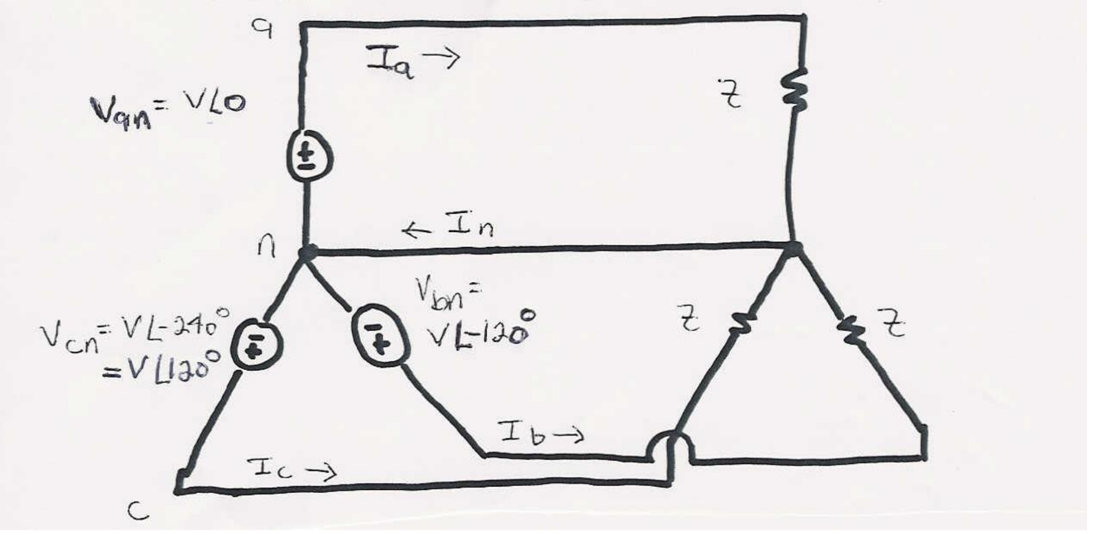
Example2

- A certain single phase load draws 5 MW at 0.7 power factor lagging. Determine the reactive power required from a parallel capacitor to bring the power factor of the parallel combination up to 0.9.

Example 3:

- A 8 MW/4 Mvar load is supplied at 13.8 kV through a feeder with an impedance of (1+ j2) . The load is compensated with a capacitor whose output, Qcap, can be varied in 0.5 Mvar steps between 0 and 10.0 Mvars. What value of Qcap minimizes the real power line losses? What value of Qcap minimizes the MVA power into the feeder?

Balanced 3ϕ -- Zero Neutral Current



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

$$S = V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^* = 3V_{an}I_a^*$$

Note: V_{xy} means voltage at point x with respect to point y.

Three Phase - Wye Connection

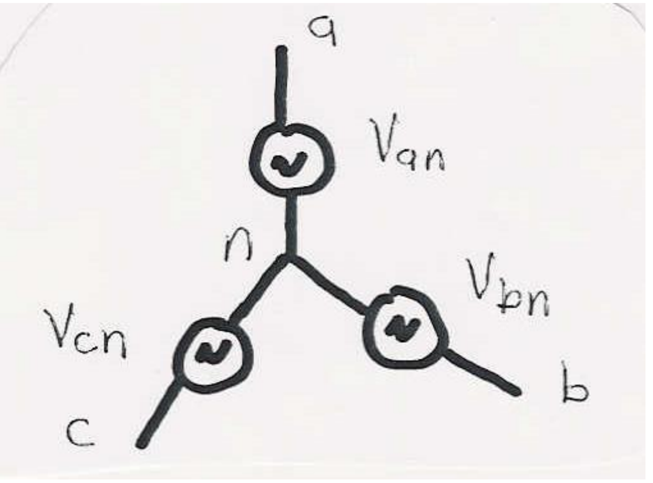
- There are two ways to connect 3ϕ systems:
  - Wye (Y), and
  - Delta (Δ).

Wye Connection Voltages

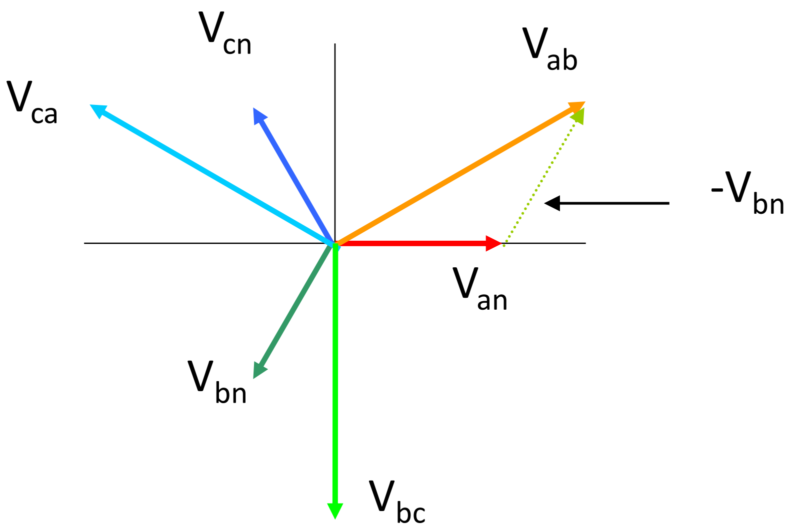
$$V_{\text{an}} = |V|\angle\alpha^\circ$$

$$V_{\text{bn}} = |V|\angle(\alpha^\circ - 120^\circ)$$

$$V_{\text{cn}} = |V|\angle(\alpha^\circ + 120^\circ)$$



Wye Connection Line Voltages



(α = 0 in this case)

$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle(\alpha - 120°))$$
$$= \sqrt{3}\,|V|\angle(\alpha + 30°)$$
$$V_{bc} = \sqrt{3}\,|V|\angle(\alpha - 90°)$$
$$V_{ca} = \sqrt{3}\,|V|\angle(\alpha + 150°)$$

Line to line voltages are also balanced.

Wye Connection, cont'd

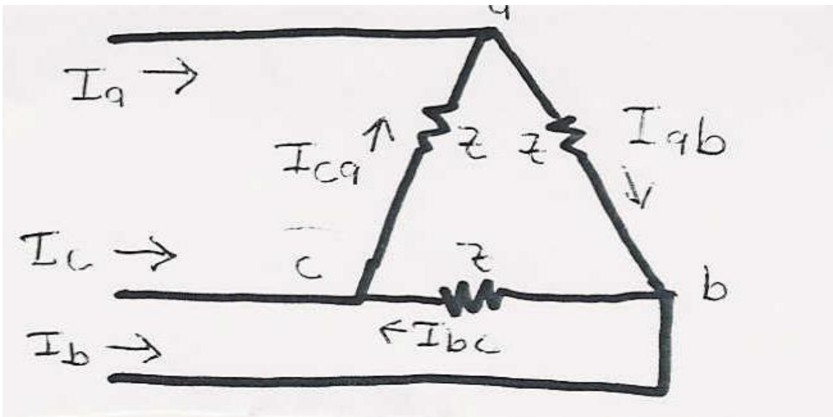
- We call the voltage across each element of a wye connected device the "phase" voltage.
- We call the current through each element of a wye connected device the "phase" current.
- Call the voltage across lines the "line-to-line" or just the "line" voltage.
- Call the current through lines the "line" current.

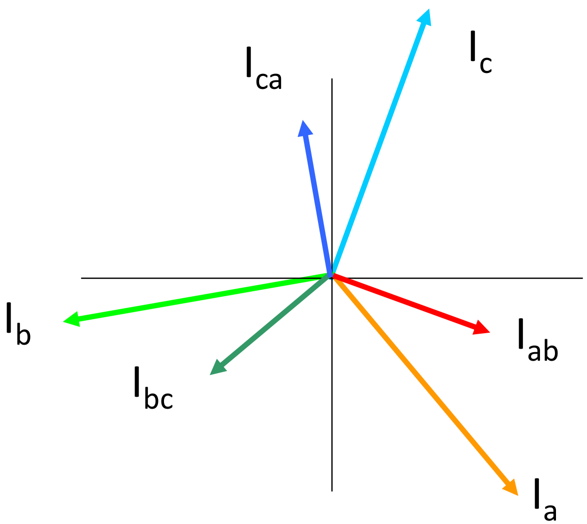
$$V_{Line} = \sqrt{3}\, V_{Phase}\, 1\angle 30^\circ = \sqrt{3}\, V_{Phase}\, e^{j\pi/6}$$

$$I_{Line} = I_{Phase}$$

$$S_{3\phi} = 3 V_{Phase}\, I^*_{Phase}$$

Delta Connection





For Delta connection, voltages across elements equals line voltages

For currents

$$I_a = I_{ab} - I_{ca}$$

$$= \sqrt{3}\, I_{ab} \angle -30^\circ$$

$$I_b = I_{bc} - I_{ab}$$

$$I_c = I_{ca} - I_{bc}$$

$$S_{3\phi} = 3 V_{Phase}\, I^{*}_{Phase}$$

Delta-Wye Transformation

To simplify analysis of balanced 3ϕ systems:

1) Δ-connected loads can be replaced by Y-connected loads with $Z_{\text{Y}} = \frac{1}{3} Z_{\Delta}$

2) Δ-connected sources can be replaced by Y-connected sources with $V_{\text{phase}} = \frac{V_{\text{Line}}}{\sqrt{3}\angle 30^\circ}$

Example 4

- A three-phase line, which has an impedance of (2 + j4) per phase, feeds a balanced Y-connected three-phase load that has an impedance of 22-4j. The line is energized at the sending end from a 50-Hz, three-phase, balanced voltage source of $230\sqrt{3}$ V (rms, line-to-line). Determine:

- The current, real power, and reactive power delivered by the sending-end source.
- The line-to-line voltage at the load.

Example5

- A industrial company has an average power consumption of 500 kW with average power factor of 0.7 and 4000 working hours. Assume that the company must not pay for reactive power if they maintain the power factor above 0.9.

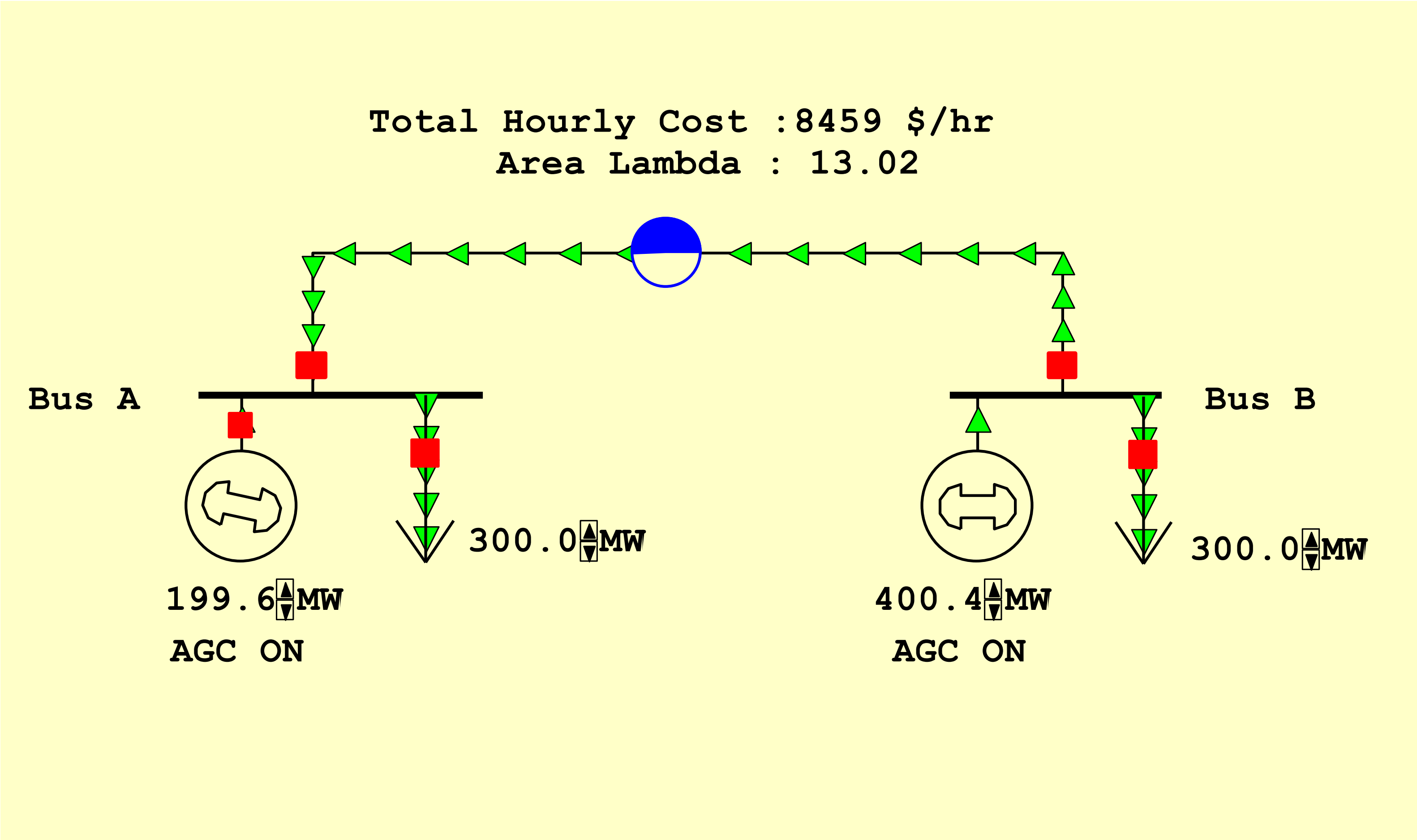
Calculate

- Annual electricity consumption (active power)
- The electricity cost for active and reactive power if 1 kWh costs 9 cents and 1 kVArh costs 1.5 cents
- Would it be profitable to install a 300 kVAr capacitor bank that costs 8000 euros?

“Ideal” Power Market

- Ideal power market is analogous to a lake. Generators supply energy to lake and loads remove energy.
- Ideal power market has no transmission constraints
- Single marginal cost associated with enforcing constraint that supply = demand
  - buy from the least cost unit that is not at a limit
  - this price is the marginal cost.
- This solution is identical to the economic dispatch problem solution.

Two Bus Example



Mathematical Formulation of Costs (C) and Incremental Cost (IC)

- Generator cost curves are usually not smooth. However the curves can usually be adequately approximated using piece-wise smooth, functions.
- Two representations predominate
  - quadratic or cubic functions
  - piecewise linear functions
- We'll assume a quadratic presentation

$$C_i(P_{Gi}) = \alpha_i + \beta P_{Gi} + \gamma P_{Gi}^2 \quad \text{\$/hr (fuel-cost)}$$

$$IC_i(P_{Gi}) = \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \quad \text{\$/MWh} \quad = \lambda$$

In order to minimize the total operating cost

$IC_1(PG_1)=IC_2(PG_2)=\ldots IC_N(PG_N)$

Mathematical Formulation of Costs

- For the two bus example

$$C(PGA) = 399.8 + 11.69PGA + 0.00334PGA^2 \text{ Euro}$$

$$C(PGB) = 616.9 + 11.83PGB + 0.00149PGB^2 \text{ Euro}$$

$$IC(PGA) = 11.69 + 0.00668PGA \text{ Euro/MWh}$$

$$IC(PGB) = 11.83 + 0.00298PGB \text{ Euro/MWh}$$

Mathematical Formulation of Costs

- For the two generator system

$$C(PGA) = 399.8 + 11.69PGA + 0.00334PGA^2 \text{ Euro}$$

$$C(PGB) = 616.9 + 11.83PGB + 0.00149PGB^2 \text{ Euro}$$

$$IC(PGA) = 11.69 + 0.00668PGA \text{ Euro/MWh}$$

$$IC(PGB) = 11.83 + 0.00298PGB \text{ Euro/MWh}$$

- PGA+PGB=600
- **λ =**IC(PGA)=13.02
- λ = IC(PGA)=13.02
- ➔PGA=200 ➔ C(PGA)=2870.6
- ➔PGB=400➔ C(PGB)=5587.3
- Total cost 8457.9

Example 6

- The fuel-cost curves for two generators are given as follows:

$$C1(P1)= 600 + 15\ P1 + 0.05\ \ P1^2$$

$$C2(P2) =700 +20P2 + 0.04\ \ P2^2$$

- Assuming the system is lossless, calculate the optimal dispatch values of P1 and P2 for a total load of 1000 MW, the incremental operating cost, and the total operating cost.